[ReMInD]
Optimal Planning of Waste Sorting Operations

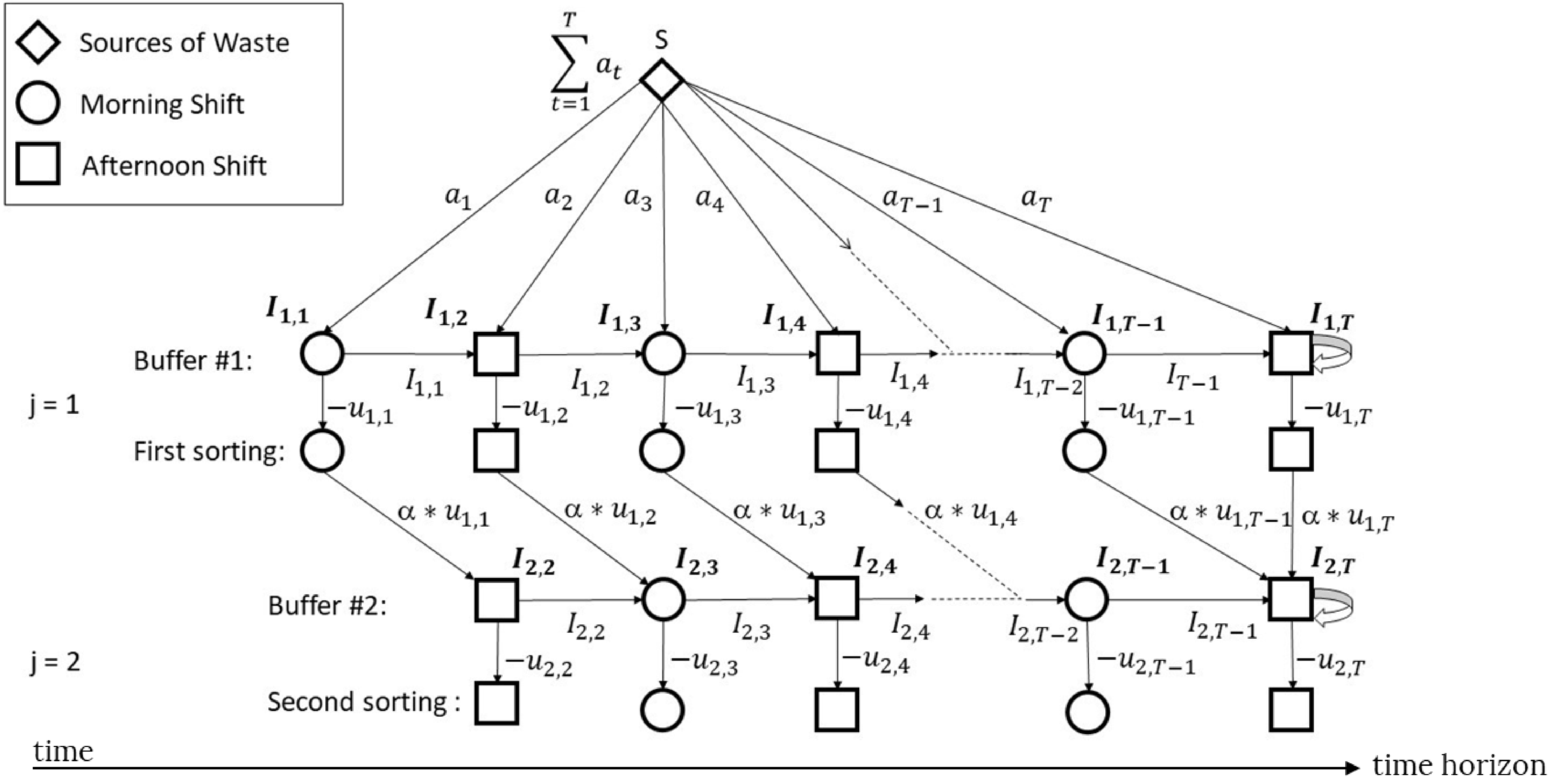
Diego Maria Pinto

Giuseppe Stecca , Claudio Gentile

CNR-IASI

diegomaria.pinto@iasi.cnr.it

A **mixed integer linear program** is used to schedule the selection operations of a two-phase waste selection process



MILP - parameters

$j = \{1, \ldots, J\}$: index of the J sorting stages
$p = \{1, \ldots, P\}$: index of the P time-shifts
T : time horizon partitioned in time shifts with $t \in \{1, \ldots, T\} = T_1 \cup \ldots \cup T_P$
C : hourly cost of each operator
σ_t : working hours for time t determined by the corresponding shift p
$C_t = C * \sigma_t$: cost of each operator at time t
f_j : set-up cost of sorting stage j
a_t : quantity of material in kg unloaded from trucks at time t
α_j : percentage of waste processed in stage $j-1$, received in input by buffer j
S_j : maximum inventory capacity of the sorting stage buffer j
LC_j : critical stock level threshold of buffer j
ρ_j : fraction of material allowed to be left at buffer j at the end of time horizon
K_j : single operator hourly production capacity [kg/h] of sorting stage j
$SK_{j,t} = K_j * \sigma_t$: operator sorting capacity in sorting stage j, at time t
M : maximum number of operators available in each time shift
E_j : minimum number of operators to be employed in each time shift of stage j
∂h_j^i : slope of the i-th part of linearization of the buffer j stock cost curve

MILP - parameters

- Time partioned in working shift

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MILP - variables

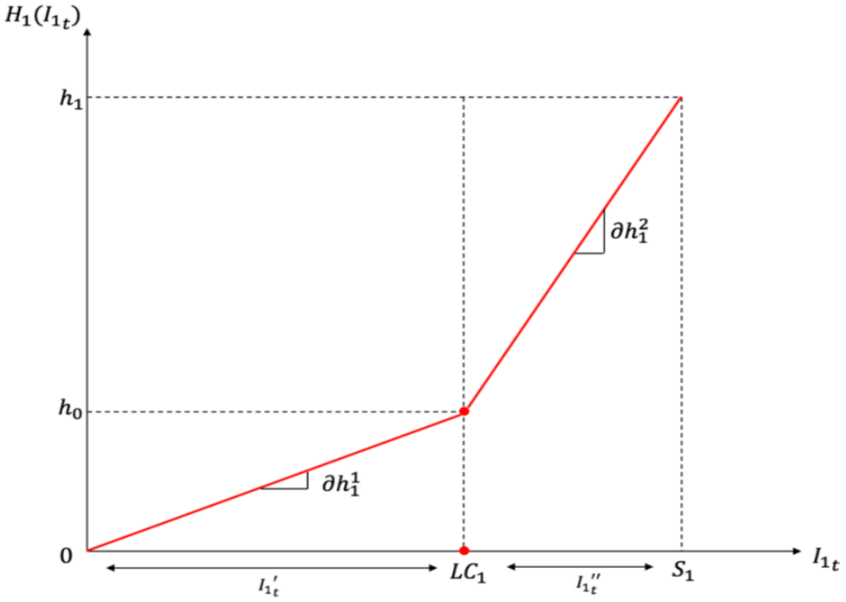
$x_{j,t} \in \mathbb{Z}^+$: operators employed in the sorting stage j at time t

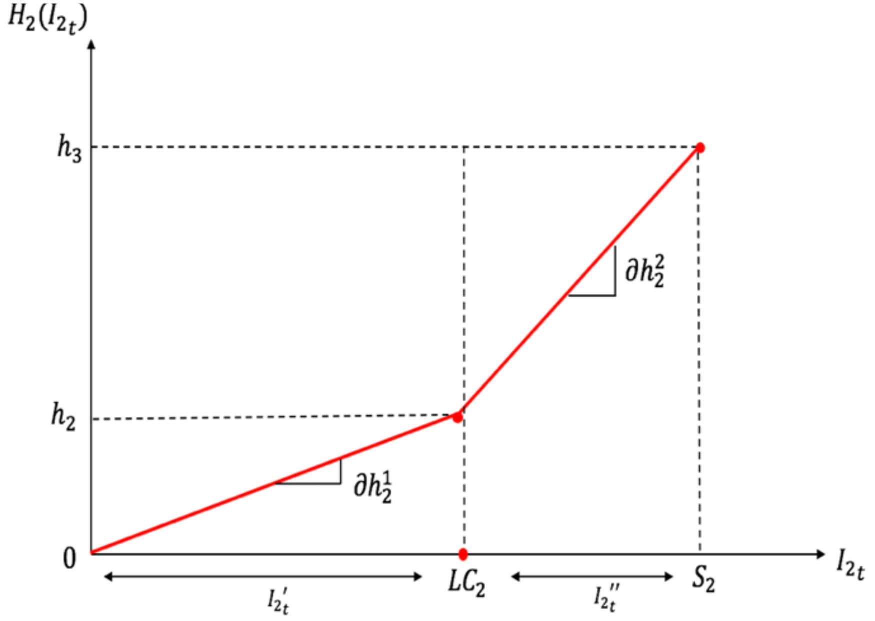
$u_{j,t} \in \mathbb{R}^+$: processed quantity at stage j at time t

$y_{j,t} \in \{0,1\}$: equal to 1 if stage j is activated at time t , 0 otherwise

$I_{j,t} = I'_{j,t} + I''_{j,t} \geq 0$: stock level of material in buffer j at time t; for each stage j the corresponding $I'_{j,t}$ and $I''_{j,t}$ represent the inventory level before and after reaching the critical threshold respectively.

$w_{j,t} \in \{0,1\}$: equal to 1 if $I''_{j,t} > 0$, 0 otherwise. Indeed, this binary variables are used to model the piece-wise linear functions of the buffer stock costs.





MILP - features

- Work shifts **scheduling**

  $y_{j,t} \in \{0,1\}$: *equal to 1 if stage j is activated at time t, 0 otherwise*

- Production **lot-sizing**

  $u_{j,t} \in \mathbb{R}^+$: *processed quantity during stage j at time t*

- **Workforce allocation**

  $x_{j,t} \in \mathbb{Z}^+$: *operators employed in sorting stage j at time t*

$$\min Z = \sum_{j\in J}\sum_{t\in T} C_t x_{j,t} + \sum_{j\in J}\sum_{t\in T} f_j y_{j,t} + \sum_{j\in J}\sum_{t\in T}\left(\partial h_j^1 I'_{j,t} + \partial h_j^2 I''_{j,t}\right) \quad (1)$$

s.t.

$$E_j\, y_{j,t} \le x_{j,t} \le M\, y_{j,t} \qquad \forall j \in J, t \in T_p, p \in P \quad (2)$$

$$\sum_{j\in J} x_{j,t} \le M \qquad \forall t \in T \quad (3)$$

$$u_{j,t} \le SK_{j,t}\, x_{j,t} \qquad \forall j \in J, t \in T \quad (4)$$

$$I_{1,t} = I_{1,t-1} + a_t - u_{1,t} \qquad \forall t \in T \setminus 0 \quad (5)$$

$$I_{j,t} = I_{j,t-1} - u_{j,t} + \alpha_j\, u_{j-1,t} \qquad \forall t \in T \setminus 0, j \in J \setminus 1 \quad (6)$$

$$I_{j,t} = I'_{j,t} + I''_{j,t} \qquad \forall j \in J, t \in T \quad (7)$$

$$LC_j\, w_{j,t} \le I'_{j,t} \le LC_j \qquad \forall j \in J, t \in T \quad (8)$$

$$0 \le I''_{j,t} \le (S_j - LC_j)\, w_{j,t} \qquad \forall j \in J, t \in T \quad (9)$$

$$I_{j,T} \le \rho_j\, LC_j \qquad \forall j \in J \quad (10)$$

$$x_{j,t} \in \mathbb{Z}^+ \qquad \forall j \in J, t \in T \quad (11)$$

$$u_{j,t} \in \mathbb{R}^+ \qquad \forall j \in J, t \in T \quad (12)$$

$$y_{j,t} \in \{0,1\} \qquad \forall j \in J, t \in T \quad (13)$$

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$$I_{1,t} = I_{1,t-1} + a_t - u_{1,t} \qquad \forall t \in T \setminus 0 \quad (5)$$

Subject to uncertainty (a_t)

$$I_{j,t} = I_{j,t-1} - u_{j,t} + \alpha_j\, u_{j-1,t} \qquad \forall t \in T \setminus 0, j \in J \setminus 1 \quad (6)$$

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MILP - ReFormulation

Constraints of stock flows regarding the first buffer consider the parameter a_t being subject to uncertainty.

These constraints are reformulated in order to match the nominal problem form of Berstsimas and Sim robustness theory.

Auxiliary variable ε_t is introduced.

Constraint (7) guarantees stock PWL costs to be included in the objective function.

s.t.

$$E_j\, y_{j,t} \le x_{j,t} \le M\, y_{j,t} \qquad \forall j \in J, t \in T_p, p \in P \qquad (2)$$

$$\sum_{j \in J} x_{j,t} \le M \qquad \forall t \in T \qquad (3)$$

$$u_{j,t} \le SK_{j,t}\, x_{j,t} \qquad \forall j \in J, t \in T \qquad (4)$$

$$I_{1,0} + \sum_{k=0}^{t} a_k \varepsilon_k - \sum_{k=0}^{t} u_{1,k} \le S_1 \qquad \forall t \in T \qquad (5)$$

$$I_{1,0} + \sum_{k=0}^{t} a_k \varepsilon_k - \sum_{k=0}^{t} u_{1,k} \ge 0 \qquad \forall t \in T \qquad (6)$$

$$I_{1,t} = I_{1,0} + \sum_{k=0}^{t} a_k \varepsilon_k - \sum_{k=0}^{t} u_{1,k} \qquad \forall t \in T \setminus 0 \qquad (7)$$

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$$I_{j,t} = I'_{j,t} + I''_{j,t} \qquad \forall j \in J, t \in T \qquad (9)$$

$$LC_j\, w_{j,t} \le I'_{j,t} \le LC_j \qquad \forall j \in J, t \in T \qquad (10)$$

$$0 \le I''_{j,t} \le (S_j - LC_j)\, w_{j,t} \qquad \forall j \in J, t \in T \qquad (11)$$

$$I_{1,0} + \sum_{t=0}^{T} a_t \varepsilon_t - \sum_{t=0}^{T} u_{1,t} \le \rho_1\, LC_1 \qquad (12)$$

$$I_{j,T} \le \rho_j\, LC_j \qquad \forall j \in J \setminus 1 \qquad (13)$$

$$\varepsilon_t = 1 \qquad \forall t \in T \qquad (14)$$

$$x_{j,t} \in \mathbb{Z}^+ \qquad \forall j \in J, t \in T \qquad (15)$$

$$u_{j,t} \in \mathbb{R}^+ \qquad \forall j \in J, t \in T \qquad (16)$$

$$y_{j,t} \in \{0,1\} \qquad \forall j \in J, t \in T \qquad (17)$$

Robust Formulation

The protection function is:

$$z_t\Gamma_t + \sum_{k=0}^{t} p_{t,k}$$

Where Γ_t is the protection parameter (robusteness lever).

$\hat{a}_t$ is the maximum deviation of parameter a_t being subject to uncertainty

Robustenss variables z_t and $p_{t,k}$ must respect constraint (31)

s.t.

$$E_j\, y_{j,t} \le x_{j,t} \le M\, y_{j,t} \qquad \forall j \in J, t \in T_p, p \in P \qquad (19)$$

$$\sum_{j \in J} x_{j,t} \le M \qquad \forall t \in T \qquad (20)$$

$$u_{j,t} \le SK_{j,t}\, x_{j,t} \qquad \forall j \in J, t \in T \qquad (21)$$

$$I_{1,0} + \sum_{k=0}^{t} a_k \varepsilon_k - \sum_{k=0}^{t} u_{1,k} + z_t\Gamma_t + \sum_{k=0}^{t} p_{t,k} \le S_1 \qquad \forall t \in T \qquad (22)$$

$$I_{1,0} + \sum_{k=0}^{t} a_k \varepsilon_k - \sum_{k=0}^{t} u_{1,k} + z_t\Gamma_t + \sum_{k=0}^{t} p_{t,k} \ge 0 \qquad \forall t \in T \qquad (23)$$

$$I_{1,t} = I_{1,0} + \sum_{k=0}^{t} a_k \varepsilon_k - \sum_{k=0}^{t} u_{1,k} + z_t\Gamma_t + \sum_{k=0}^{t} p_{t,k} \qquad \forall t \in T \setminus 0 \qquad (24)$$

$$I_{j,t} = I_{j,t-1} - u_{j,t} + \alpha_j\, u_{j-1,t} \qquad \forall t \in T \setminus 0, j \in J \setminus 1 \qquad (25)$$

$$I_{j,t} = I'_{j,t} + I''_{j,t} \qquad \forall j \in J, t \in T \qquad (26)$$

$$LC_j\, w_{j,t} \le I'_{j,t} \le LC_j \qquad \forall j \in J, t \in T \qquad (27)$$

$$0 \le I''_{j,t} \le (S_j - LC_j)\, w_{j,t} \qquad \forall j \in J, t \in T \qquad (28)$$

$$I_{1,0} + \sum_{k=0}^{T} a_k \varepsilon_k - \sum_{k=0}^{T} u_{1,k} + z_T\Gamma_T + \sum_{k=0}^{T} p_{T,k} \le \rho_1\, LC_1 \qquad (29)$$

$$I_{j,T} \le \rho_j\, LC_j \qquad \forall j \in J \setminus 1 \qquad (30)$$

$$z_t + p_{t,k} \ge \hat{a}_t\, s_t \qquad \forall t \in T, k \in \{0, \dots, t\} \qquad (31)$$

$$-s_t \le \varepsilon_t \le s_t \qquad \forall t \in T \qquad (32)$$

$$\varepsilon_t = 1 \qquad \forall t \in T \qquad (33)$$

$$x_{j,t} \in \mathbb{Z}^+ \qquad \forall j \in J, t \in T \qquad (34)$$

$$u_{j,t} \in \mathbb{R}^+ \qquad \forall j \in J, t \in T \qquad (35)$$

$$y_{j,t} \in \{0,1\} \qquad \forall j \in J, t \in T \qquad (36)$$

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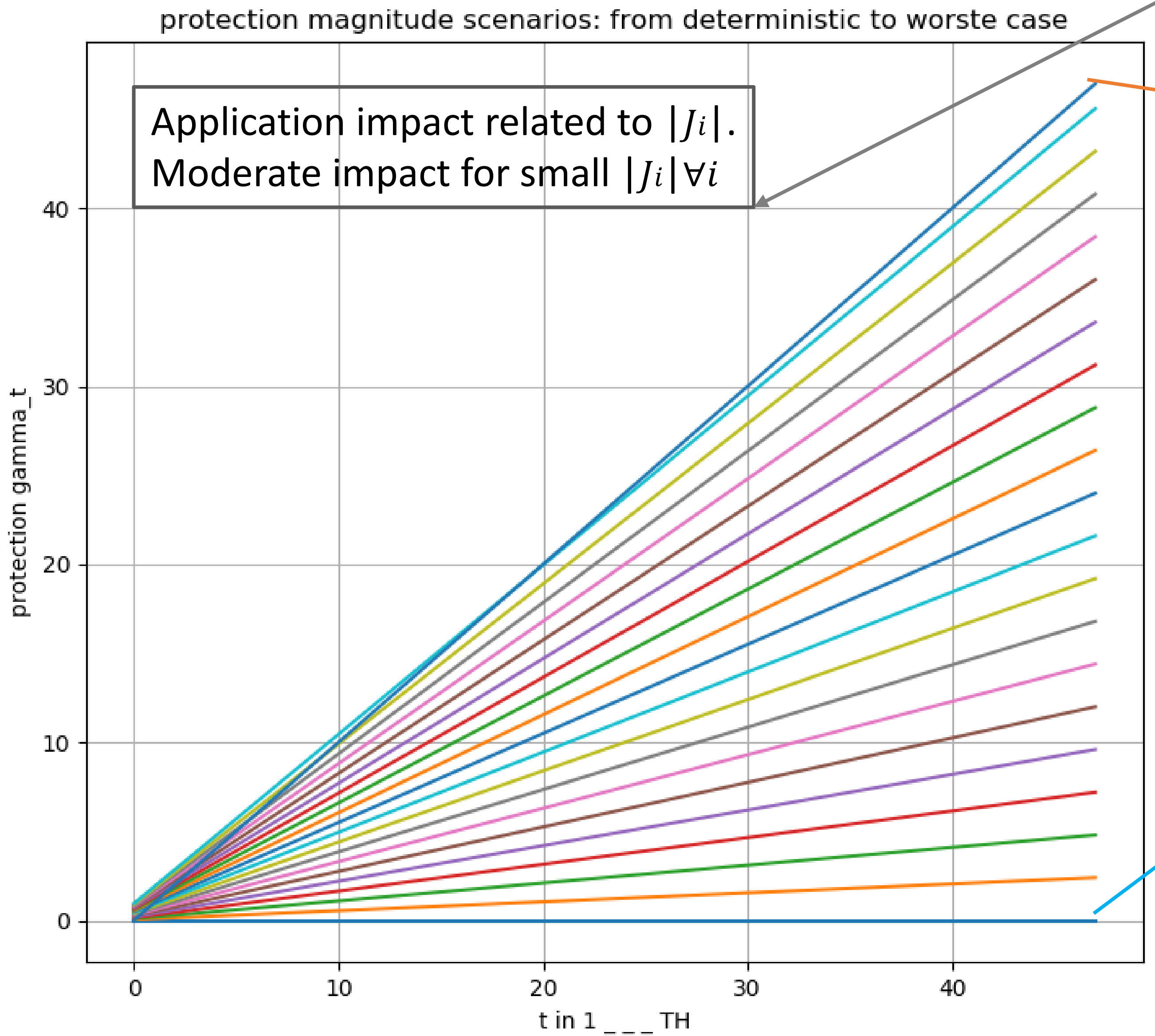
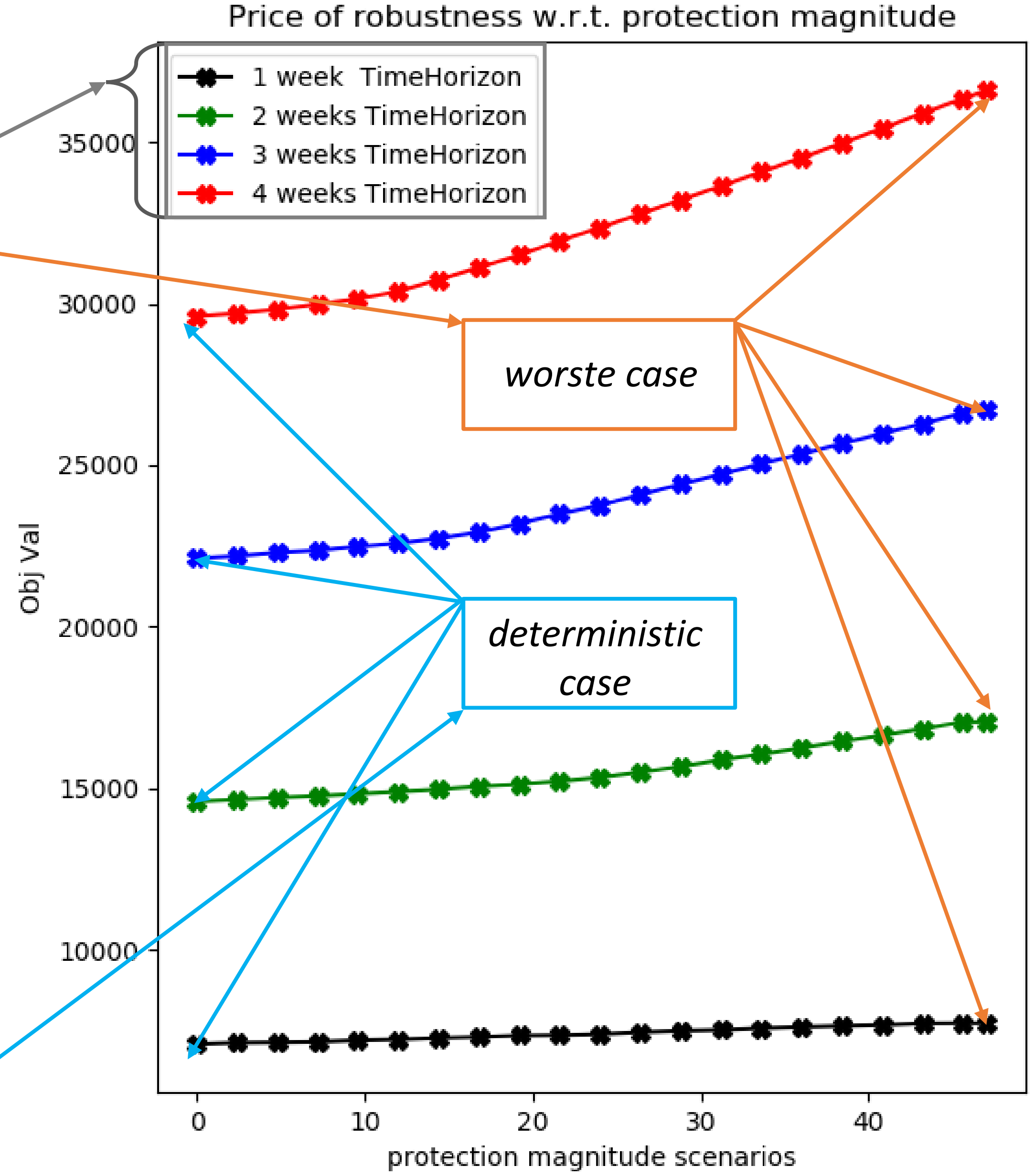
the price of robustness





Current and future works

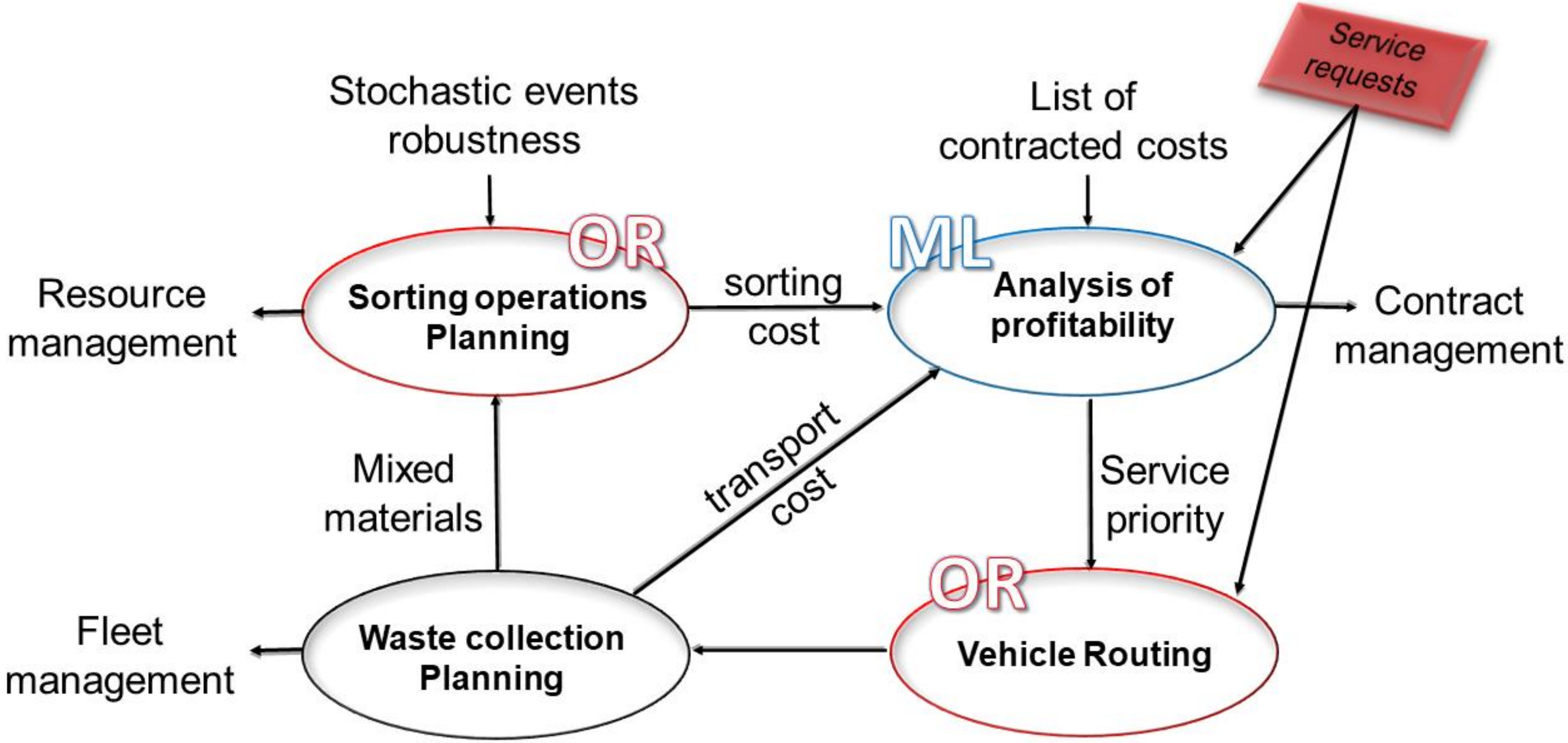
- Enlarge experiments and considerations regarding robustness and probabilistic bounds over constraint violations. Considering **correlation between parameters** that are subject to uncertainty.

- Develop a production model version that includes **waste baling** press operations

- Considering **production capacity as a function of operators employed**

- **Profit patterns recognition** trough logistic and sorting models integration: **framework of models integration** for profitability analysis and contract management support.

Models Integration Framework



SLWA := Scheduling with Lot sizing and Workforce Allocation
PDTW := Pickup and Delivery with Time Windows

Models Integration information flow diagram for profit patterns dataset creation